Bapuji Educational Association (Regd.)
Bapuji Institute of Engineering and Technology, Davangere - 04

DEPARTMENT OF MATHEMATICS

Question Bank

TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES (18MAT31)

1. $L\left[e^{-2t}(2\cos 5t-\sin 5t)+\cos^2 3t\right]$
2. $L\left[\frac{\cos 2t-\cos 3t}{t}+t\,sint+2\sin 3t\cos 5t\right]$
3. Given $f(t)=\begin{cases} k, & 0<t<a/2 \\ -K, & a/2<t<a \end{cases}$ where f(t+a) =f(t) Show that $L[f(t)]=\frac{k}{s}\tanh$ (sa/4)
4. A periodic function of period $\frac{2\pi}{\omega}$ is defined by $f(t)=\begin{cases} E\sin\omega t & 0\le t<\pi/\omega \\ 0 & \pi/\omega\le t<2\pi/\omega \end{cases}$ find L[f(t)]
5. Express $f(t)=\begin{cases} 1 & 0<t\le 1 \\ t & 1<t\le 2 \\ t^2 & t>2 \end{cases}$ in terms of unit step function and hence find its Laplace transform.
6. Express $f(t)=\begin{cases} \cos t, & 0<t\le\pi \\ 1, & \pi<t\le 2\pi \\ \sin t, & t>2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform.
7. Find $L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}+\log\left(\frac{s+a}{s+b}\right)+\frac{s+1}{s^2+6s+9}\right]$
8. By using convolution theorem, find $L^{-1}\left[\frac{1}{(s^2+1)(s-1)}\right]$
9. By using convolution theorem, find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$
10. Solve $y^{II}+4y^{I}+4y=e^{-t}$ with the initial conditions $y(0)=y^{I}(0)=0$
11. Obtain the Fourier series of $f(x)=\frac{\pi-x}{2}$ in $0<x<2\pi$. hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots=\frac{\pi}{4}$
12. Obtain the fourier series of $f(x)=\begin{cases} -\pi, & -\pi<x<0 \\ x, & 0<x<\pi \end{cases}$
13. Obtain the Fourier series of $f(x)=x-x^2$ in $-1<x<1$
14. Obtain the fourier series of $f(x)=\begin{cases} \pi x, & 0\le x\le 1 \\ \pi(2-x), & 1\le x\le 2 \end{cases}$
15. Obtain the half range sine series of $f(x)=x^2$ in $0<x<\pi$

16. Obtain the half range cosine series of $f(x)=¿$ in [0.1]

17. Given the following table

x^0	0	60	120	180	240	300
y	7.9	7.2	3.6	0.5	0.9	6.8

Obtain the Fourier series neglecting terms higher than first harmonics

18. Obtain the constant term and coefficient first cosine and sine terms in the Fourier expansion of y from the following table

x	0	1	2	3	4	5
y	9	18	24	28	26	20

19. Obtain the constant term and coefficient of s$sin\theta \wedge \sin 2\theta$in the Fourier expansion of y from the following table

x^0	0	60	120	180	240	300	360
y	0	9.2	14.4	17.8	17.3	11.7	0

20. The following data gives the variations of periodic current over a period.find the direct current part of the variable current and obtain the amplitude of the first harmonic

t sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A amps	9.0	18.2	24.4	27.8	27.5	22.0	9.0

21. Find the Fourier transform of $f(x)=\begin{cases}1\, for\, |x|\le a\\ 0\, for\, |x|>a\end{cases}$ and hence deduce that $\int_0^{\infty}\frac{sinx}{x}dx=\frac{\pi}{2}$

22. Obtain Fourier Sine transform of $f(x)=¿\,e^{-|x|}$. Hence show that $\int_0^{\infty}\frac{xsinmx}{1+x^2}dx=\frac{\pi e^{-m}}{2}$, m¿0

23. Find the Fourier Transform for the function $f(x)=\begin{Bmatrix}1\, for\, |x|\le a\\ 0\, for\, |x|>a\end{Bmatrix}$ and hence evaluate

$$\int_0^{\infty}\frac{sinx}{x}dx=\frac{\pi}{2}$$

24. Find the Fourier Sine and Cosine Transform of $f(x)=\begin{Bmatrix}x\, for\, 0<x<2\\ 0\, for\, Otherwise\end{Bmatrix}$

25. Prove that $(a)\, Z_T(n^k)=-z\frac{d}{dz}Z_T(n^{k-1})\,(b)\, Z_T(n^2)$

26. Find the Z-transform of (i) $\cosh n\theta$ (ii) $\sinh n\theta$

27. Given $Z_T(U_n)=\frac{2z^2+3z+4}{(z-3)^3}, |z|>3$ Find $U_0, U_1 \wedge U_2$

28. Find the Inverse Z-transform of $Z_T^{-1}\left[\frac{5z}{(2-z)(3z-1)}\right]$

29. Solve the Difference equation $U_{n+2}-5U_{n+1}+6U_n=0$ by using Z-transform

30. Find the Z-transform of (i) $\cos n\theta$ (ii) $\sin n\theta$

31. Given $Z_T(U_n)=\frac{2z^2+3z+4}{(z-3)^3}, |z|>3$ Find $U_0, U_1 \wedge U_2$

32. Use Taylor's series method to solve $\frac{dy}{dx}=2y+3e^x, y(0)=0$ *find y at* $x=0.2$

33. Find by Taylor's series method the value of y at x = 0.1 and x = 0.2 up to five places of decimals from $\frac{dy}{dx}=x^2y-1$, y (0) =1, (consider upto fourth degree terms).

34. Use Taylor's series method the value of y at x = 0.1 and x =0.2 to four decimal places from $\frac{dy}{dx}=x^2+y^2$, $y(0)=1$, consider up to Third degree terms.

35. Use Modified Euler's method to find an approximate value of y when x = 0.2 given that $\frac{dy}{dx}=3x+\frac{y}{2}\wedge y(0)=1$.take h = 0.1. Perform three iterations in each stage.

36. Given $\frac{dy}{dx}=3x+\frac{y}{2}, y=1$ *at* $x=0$. Use Modified Euler's method to find y(0.2) taking h = 0.2

37. Using Modified Euler`s method to find y(20.2) and y(20.4) given that $\frac{dy}{dx}=\log_{10}\left(\frac{x}{y}\right)$ with y(20)=5 taking h=0.2

38. Given $\frac{dy}{dx}=x(y)^{\frac{1}{3}}, y(1)=1$. Use Runge-Kutta fourth order method to find y at x = 1.1

39. Given $\frac{dy}{dx}=x^2(1+y)\wedge y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979$
evaluate y(1.4) by Milne's Predictor-Corrector method

40. Given $\frac{dy}{dx}=x-y^2\wedge y(0)=0, y(0.2)=0.02, y(0.4)=0.0795, y(0.6)=0.1762$
Compute y at x= 0.8 by Milne's Predictor-Corrector method.

41. Given $\frac{dy}{dx}=x^2(1+y)$, $y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979$ determine y(1.4) by Adams-Bashforth method. Use Corrector formula twice.

42. Given $\frac{dy}{dx}=x-y^2$ and the data $y(0)=0, y(0.2)=0.02, y(0.4)=0.0795, y(0.6)=0.1762$ Compute y at x=0.8 by applying Adams-Bashforth method.

43. Use Modified Euler's method to find an approximate value of y when x = 0.2 given that $\frac{dy}{dx}=x+y\wedge y=1$ *when* $x=0$.take h = 0.1. Perform two iterations in each stage

44. Use Runge-Kutta method of fourth order solve $y^{11}-xy^1-y=0, y(0)=1, y^1(0)=0$
find y and z at x = 0.2

45. Use Runge-Kutta method of fourth order solve $y^{11}-x^2y^1-2xy=1, y(0)=1, y^1(0)=0$
Evaluate y(0.1)

46. Given $y^{11}=y^3, y(0)=10, y^1(0)=5$.Evaluate $y(0.1)$using Runge-Kutta method of fourth order.

47. Apply Milne's method to solve $\frac{d^2 y}{dx^2}=1-2y\frac{dy}{dx}$, $given\ y(0)=0, y^1(0)=0$.compute $y(0.8)$ given

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y^1	0	0.1996	0.3937	0.5689

48. Given $y^{11}-x^2y^1-2xy=1$ with the initial conditions y(0) = 1,$y^1(0) = 0$, compute y(0.1) using fourth order Runge-Kutta method.

49. Apply Milne`s method to compute y(0.8) given that

$\frac{d^2 y}{dx^2}=1-2y\frac{dy}{dx}$ and the following table of initial values

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
Y^1	0	0.1996	0.3937	0.5689

50. State and prove Euler's Equation